

Condition for the burning of hadronic stars into quark stars

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1 Introduction

The question about the existence of absolutely stable strange quark matter in astrophysical compact objects is still debated in a number of theoretical and observational investigations, see Ref. [1] for a recent review. In particular the formation of quark stars is a very interesting issue. In the seminal paper [2], the process of conversion of a neutron star into a quark star was shown to be a very exothermic process which, if it really occurs in Nature, should lead to powerful explosive phenomena, maybe also connected with supernovae and gamma-ray-bursts [3, 4, 5]. The relevance of this process, from the phenomenological point of view, is related to the time scale of the conversion. Only if sufficiently fast (with time scales of the order of tens of seconds or less) the formation of a quark star could release detectable signals; on the other hand, a very slow conversion would not probably provide any evident signature. Interestingly, semi-analytical estimates and numerical hydrodynamics simulations have provided hints for a very fast conversion, occurring on time scales of the order of ms [6, 7, 8, 9, 10]. In this short contribution, we will review the approach used for studying the conversion of a neutron star into a quark star based on the assumption of a infinitely thin combustion zone and we will discuss why, in this scheme, the combustion stops before the whole hadronic star is converted.

2 Combustion of hadronic stars

The process of combustion is a very complicated phenomenon which in principle must be modeled by coupling the hydrodynamic equations for the mixture of the two fluids (the fuel and the ashes) with the equations of conservation of chemical species in which both diffusion processes and rates of chemical reactions are included [11], see [12, 8] for the case of quark stars. A common assumption, especially used in astrophysical problems such as type Ia supernovae [13], consists in treating the combustion zone as a surface of discontinuity separating the fuel from the ashes. This assumption is necessary since in many cases the width of the combustion zone is much smaller then the size of the system and it would be computationally unfeasible

to resolve the microscopic dynamics of the combustion zone. As in the case of shock waves, in presence of a discontinuity, hydrodynamic equations lead to the conditions of continuity of the baryon flux, the momentum flux and the energy flux across the burning front [14]. Indicating with e_i , p_i , n_i , w_i , $X_i = (e_i + p_i)/n_i^2$ the energy density, pressure, baryon density, enthalpy and dynamical volume of the i -th fluid, those conservation laws, generalized to relativistic hydrodynamics [15], read:

$$n_1 u_1 = n_2 u_2 = j \quad (1)$$

$$(p_2 - p_1)/(X_1 - X_2) = j^2 \quad (2)$$

$$w_1(p_1, X_1)X_1 - w_2(p_2, X_2)X_2 = (p_1 - p_2)(X_1 + X_2) \quad (3)$$

where u_i are the four-velocities of the two fluids in the front rest frame and j is the baryon number flux across the front and it must be determined within a microscopic kinetic approach, as the one of Ref. [2]. This system of equations allows to determine the velocities of the two fluids and the state of fluid 2 once the initial state of fluid 1 is fixed. Depending on the values of the velocities u_i and the sound velocities of the fluids c_i , one can obtain detonations, which are processes of combustion driven by a shock wave, or deflagrations in which instead combustion proceeds thanks to the diffusion of heat or of chemical species [14].

The condition of exothermic combustion, generalized to relativistic hydrodynamics by Coll in Ref. [15], reads $\Delta(p, X) = e_1(p, X) - e_2(p, X) = w_1(p, X) - w_2(p, X) > 0$ and allows to find the window of baryon densities of fluid 1 for which the combustion can proceed. We want here to clarify the meaning of this condition. Let us fix the initial state A of fluid 1: $p_1 = p_A$, $e_1 = e_A$, $X_1 = X_A$ (the temperature of fluid 1 is set to zero). From this point one can draw the shock adiabat of fluid 1 in the (p, X) plane. Moreover, by using the equation of state of fluid 2 and Eq.3 one can draw in the same plane the detonation adiabat, see Fig.1. In the following we will demonstrate that, the Coll's condition implies that the shock adiabat lies below the detonation adiabat. Let us assume that the equation of state of fluid 2 is a generic polytrope $e_2 = \alpha n_2 + p_2/(\gamma - 1)$ ([16]), $p_2 = k n_2^\gamma$, where γ is the adiabatic index and $1 < \gamma \leq 2$ (the second inequality implying that the equation of state is causal at all densities [17]).

One can derive the following expression for the energy density as a function of p_2 and X_2 :

$$e_2(p_2, X_2) = \frac{\alpha^2(\gamma - 1) + 2p_2X_2 + \alpha\sqrt{\gamma - 1}\sqrt{\alpha^2(\gamma - 1) + 4\gamma p_2X_2}}{2X_2(\gamma - 1)} \quad (4)$$

Let us fix $X_1 = X_2 = X_A$ and assume $\Delta(p_A, X_A) > 0$. If $p_2 > p_1 = p_A$, the detonation adiabat lies above the shock adiabat. With this setting, the detonation adiabat reads (adding and subtracting $e_2(p_A, X_A)$):

$$\Delta(p_A, X_A) = p_A - p_2 + e_2(p_2, X_A) - e_2(p_A, X_A) \quad (5)$$

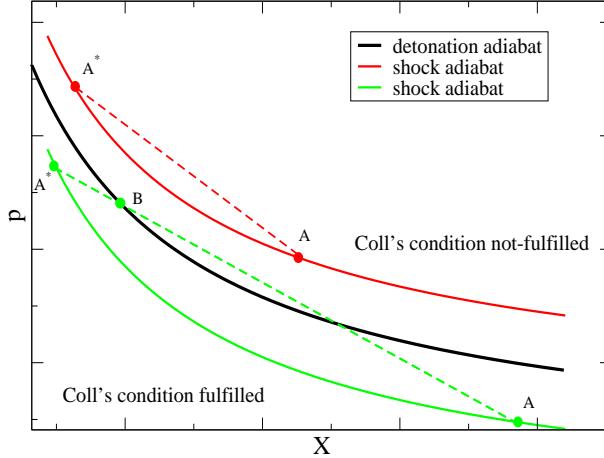


Figure 1: Illustrative plot of the shock and the detonation adiabats in the case in which the Coll's condition is fulfilled or not.

which after some manipulation and using Eq.4 reads:

$$\Delta(p_A, X_A) = \frac{\alpha}{2X_A\sqrt{\gamma-1}} \left(\sqrt{\alpha^2(\gamma-1) + 4\gamma p_2 X_A} - \sqrt{\alpha^2(\gamma-1) + 4\gamma p_A X_A} \right) \quad (6)$$

$$+ (p_2 - p_A) \frac{2-\gamma}{\gamma-1} \quad (7)$$

Since $1 < \gamma \leq 2$ the sign of $\Delta(p_A, X_A)$ clearly determines the sign of $p_2 - p_A$. Thus, if $\Delta(p_A, X_A) > 0$, i.e. if the Coll's condition holds true, the initial point A lies below the detonation adiabat ¹. Since the detonation adiabats and the shock adiabats do not cross in the (p,X) plane [14] this implies that the whole shock adiabat lies below the detonation adiabat (at least in the standard cases). The Coll's condition is necessary for obtaining detonations. Detonation is a process of combustion which is driven by a shock wave propagating within the fuel: when the shock wave passes through A, the fluid is compressed and heated up to the state A* which lies on the shock adiabat (see Fig.1). The chemical reactions start, the fluid expands and cools down until the combustion is complete and the state B, lying on the detonation adiabat, is reached [14]. On the other hand, if the shock adiabat of the fuel lies above the detonation adiabat, it is not possible to trigger the combustion via a shock wave. A shock wave passing through the point A would heat up the matter but not at a sufficiently high temperature to start the combustion (see Fig.1). Once the Coll's condition is fulfilled the hydrodynamical combustion can proceed either as a detonation or a deflagration

¹For values of $\gamma > 2$, one can still obtain the same conclusion but depending on the specific values of α , p_A and X_A

depending on the specific microphysics and on the equation of state. The equation $\Delta(p_A, X_A) = 0$ allows to find the state A, characterized by its baryon density n_1^* , below which the combustion cannot proceed anymore. At this density $e_1 = e_2 = e_A$, $p_1 = p_2 = p_A$ which together with $X_1 = X_2 = X_A$ also implies $n_1 = n_2 = n_1^*$. At this value of baryon density there is no surface of discontinuity anymore and the two phases are in mechanical equilibrium. Previous studies have shown that, for many equations of state, $n_1^* \sim 0.2 - 0.3 \text{ fm}^{-3}$ [6, 9] and indeed the fast combustions found in the numerical simulations of Ref. [9] stop exactly at those values of density.

One has to remind that the scheme based on a infinitely thin combustion zone is only an approximation. Relieving this approximation and considering the microscopic processes of diffusion of quarks and reactions between quarks (such as $u+d- > u+s$) occurring in the finite width combustion zone allows to follow the subsequent evolution of the system which proceeds until the whole star is converted [18].

3 Conclusions

We have discussed the approximation scheme of combustion based on the assumption of a infinitely thin combustion layer. This model has been widely used in numerical simulations of type Ia supernovae [13] and recently also for numerical investigations of the conversion of hadronic stars into quark stars [9, 10]. At densities larger than the critical density n_1^* , for which the Coll's condition is fulfilled, the process of conversion proceeds very fast with the effective velocity of conversion significantly augmented by Rayleigh-Taylor instabilities. After a few ms, when the conversion front reaches n_1^* , a big part of the star is converted but a few $0.1 M_\odot$ remain unburnt and will convert on a longer time scale, of the order of tens of seconds [18]. Note that, up to now, the conversion process has been studied only within cold and non rotating hadronic stars. The dynamics of the birth of quark stars can be qualitatively different in other astrophysical situations such as supernovae and protoneutron stars and mergers of neutron stars. Those cases are, to date, essentially unexplored and detailed investigations are therefore needed to better clarify the scenario of coexistence of two families of compact stars, hadronic stars and quark stars, proposed in [19, 20].

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